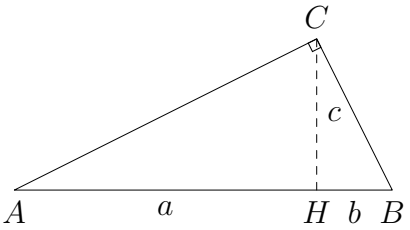
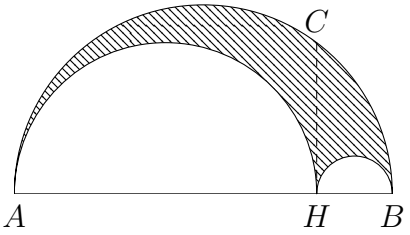
PERSONAL TUTORIAL 4.

1. Question Sheet 1

Qu 1. a. Consider a right-angled triangle as in the figure below. Show that $ab = c^2$.



b. Compute the area of the shaded region enclosed by the three half circles of diameter AH, HB, and AB. We only know that the length of CH is $\sqrt{3}$. (To avoid confusion: AB is horizontal and H is the intersection of the vertical line through H with the half circle of diameter AB.)



Qu 2. Consider a 10×10 chessboard filled with the numbers from 1 to 100 as in the figure below:

	a	b	c	d	e	f	g	h	i	j
10	91	92	93	94	95	96	97	98	99	100
9	81	82	83	84	85	86	87	88	89	90
8	71	72	73	74	75	76	77	78	79	80
7	61	62	63	64	65	66	67	68	69	70
6	51	52	53	54	55	56	57	58	59	60
5	41	42	43	44	45	46	47	48	49	50
4	31	32	33	34	35	36	37	38	39	40
3	21	22	23	24	25	26	27	28	29	30
2	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10

Suppose now that we change sign to 50 of these numbers, so that every row and every column contains 5 negative and 5 positive numbers. Show that, after such change of signs, the sum of all numbers on the chessboard is 0.

Qu 3. **Extra question if times remains:** Let n be a positive integer and let

$$f(n) = \frac{5n + 97}{n + 7}.$$

How many positive integers n are there, such that $f(n)$ is an integer?

References

[1] R. Fitzpatrick and J. Heiberg. (2007) Euclid's elements. [Online]. Available: `http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf`

2. Question Sheet 2

Qu 1. **(MATH40005 Probability and Statistics)**

The newspaper Guardian reported in an article entitled "The rise of lateral flow tests: are these 'heroes' of the pandemic here to stay?" on 7th January 2022 the following:

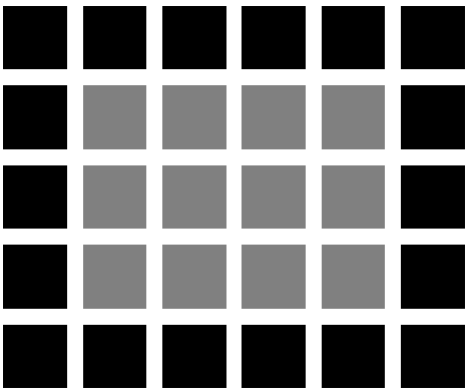
> "As case numbers fell in 2021, lateral flow tests also came under criticism for delivering false positives. This reflects a classic statistical conundrum: if you test positive, the chances that you really have Covid depends on not only the quality of the test but also the current prevalence of the virus. For instance, if an LFT delivers a false positive result for one in 1,000 tests at a time when just one in 1,000 people have Covid, the chances that someone with a positive [result] is truly infected are only 50%. But if the prevalence is one in 10 people, as it was in London last week, that person can be 99% sure they are infected."

Note that LFT stands for lateral flow test.

Can you verify the statement above using results from the lectures? Did the journalist make any implicit assumptions?

Qu 2. **(MATH 40006 Introduction to Computations)**

The diagram shows a 5×6 rectangle. There are 12 **internal** squares, shown in grey. There are 18 **external** squares, shown in black. What are the possible integer rectangle dimensions (width and height) for which the numbers of internal and external squares are equal?



Extending this to three dimensions, what are the possible integer *cuboid* dimensions (width, depth and height) for which the numbers of internal and external *cubes* are equal?

Hint: This "cuboid" version of the problem has quite a large number of solutions, and we recommend that you write a short piece of Python code to list them all. But try not to "brute-force" it; use the insights from your solution to the "rectangle" version to help you solve the "cuboid" problem efficiently.