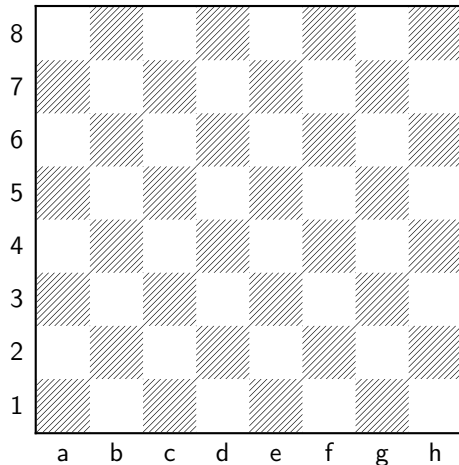


PERSONAL TUTORIAL 1.

- Qu 1. Suppose we remove two diagonally opposite corners from an 8×8 chessboard. Is it possible to cover all the remaining 62 squares with 31 2×1 domino pieces? (The domino pieces have to lie entirely on the surface formed by the 62 remaining squares)



- Qu 2. Ivor Vineyard is a wine producer, but not a very honest one. Ivor wants to water down the wine to increase profit. Vineyard has two barrels A and B , containing water and wine respectively. Both A and B contain the same amount x of liquid. Ivor takes a cup of water from A and pours it into B . Then, to level the barrels, Ivor takes a cup of the new mixture from B and puts it back into A .

- (a) How does the amount of wine in A compare with the amount of water in B ?

Hint: Think of what happens if the cup has volume 0 or volume x . What is your guess?

- (b) Make an educated guess on what happens if the process is repeated infinitely many times.

Hint: Think of what happens if we start with $w = x/2$.

- (c) What happens in general if, at the start, the liquid in A is a mixture containing (in volume) w of wine and $x - w$ of water, while B contains $x - w$ of wine and w of water?

- (d) Write a recursive formula giving the amount of water in B after n repetitions of the process described in the question.

- Qu 3. (**Extra question if time remains**) Wildlife photographer Greta Picture travels to the remote island of *Salamandria*. On this island, she finds exactly 45 salamanders: 17 red ones, 13 yellow ones, and 15 green ones. Every time two salamanders of different colours meet, they turn the third colour. Is it possible Greta takes a picture, in which all the 45 salamanders are the same colour?

Extra: What happens in general for $3n$ total salamanders r of which red, y of which yellow and g of which green?